2018 Annual American Control Conference (ACC)
June 27–29, 2018. Wisconsin Center, Milwaukee, USA

Translational Maneuvering Control of Nonholonomic Kinematic Formations: Theory and Experiments

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Abstract— **In this work, we present a solution to the distance-based formation maneuvering problem of multiple nonholonomic unicycle-type robots. The control law is designed at the kinematic level and is based on the rigidity properties of the graph modeling the sensing/control interactions among the robots. A simple input transformation is used to facilitate the control design by converting the nonholonomic model into the single-integrator equation. The resulting control ensures exponential convergence to the desired formation while the formation maneuvers according to a desired, time-varying translational velocity. An experimental implementation of the proposed control law is conducted on the Robotarium testbed.**

I. Introduction

The field of decentralized control of multi-agent systems is an ongoing topic of interest to control and robotics researchers. Formation control is a type of coordinated behavior where mobile agents are required to autonomously converge to a specified spatial pattern. Many coordinated tasks, such are target interception, element tracking, and exploration, also require that the formation maneuver as a virtual rigid body. Such maneuvers can include translation, rotation, or the combination of both.

Formation control algorithms have been designed for different models of the agent motion. Most results are based on point-mass type models, such as the single- and double-integrator models. For example, see [19], [29], [47] for single-integrator results and [10], [11], [36], [44] for double-integrator results. On the other hand, some results have used more sophisticated models that account for the agent kinematics/dynamics. One of two models are used in these cases: the fully-actuated (holonomic) Euler-Lagrange model, which includes robot manipulators, spacecraft, and some omnidirectional mobile robots; or the nonholonomic (underactuated) model, which accounts for velocity constraints that typically occur in the vehicle motion (e.g., differentially-driven wheeled mobile robots and air vehicles). In the nonholonomic case, models can be further subdivided into two categories: the purely kinematic model where the control inputs are at the velocity level, and the dynamic model where the inputs are at the actuator level. Examples of work based on the Euler-Lagrange model include [9], [12], [14], [27], [30], [37], [45], [46]. Formation control results based on nonholonomic kinematic models can be found in [5], [33], [35], [41]. Designs for nonholonomic dynamic models appeared in [13], [17], [18], [20], [31].

Translational maneuvering and target interception controllers were introduced in [8], [7] for the single- and double-integrator models, respectively, using the distance-based, rigid graph approach from [29]. A 2D formation maneuvering controller was proposed in [4] for the double-integrator model where the group leader, who has inertial frame information, passes the information to other agents through a directed path in the graph. A limitation of this control is that it becomes unbounded if the desired formation maneuvering velocity is zero. In [25], a leader-follower type solution was given to the formation maneuvering problem based on the nonholonomic kinematics of unicycle robots. A consensus scheme was presented in [22] using both the single- and double-integrator models where the formation translation velocity is constant and known to only two leader agents. In [40], the translational maneuvering strategy involved a leader with a constant velocity command and followers who track the leader while maintaining the formation shape. The control law, which was based on the single-integrator model, consisted of the standard gradient descent formation acquisition term plus an integral term to ensure zero-steady error with respect to the velocity command. In [43], for agents modeled by double integrators, a flocking controller was designed that allows all agents to both achieve the same velocity and reach a desired formation in finite time. A similar problem was addressed in [15] but with asymptotic formation acquisition and velocity consensus. Recently in [32], a controller was proposed using the single-integrator model that can steer the entire formation in rotation and/or translation in 3D. The rotation component was specified relative to a body-fixed frame whose origin is at the centroid of the desired formation and needs to be known.

In formation control, a key aspect is whether the controlled variables are the relative position vector of the agents or the inter-agent distances (i.e., norm of the relative position). The latter approach has the advantage that relative position measurements can be done in an arbitrary coordinate frame, whereas the former requires the measurements in a global coordinate frame [42]. The distance-based control framework has been mostly applied to the single- and double-integrator models. To the best of our knowledge, the only exception are the results in [16], [48] which considered the nonholonomic kinematic model in the design of a formation acquisition controller. In this paper, we apply the distance-based approach to the translational maneuvering of nonholonomic kinematic

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978-1-5386-5428-6/$31.00 ©2018 AACC

agents in the form of unicycle-type vehicles. Similar to [16], we use a simple input transformation to convert the nonholonomic multi-agent system into the single-integrator system. As a result, we can apply the formation maneuvering controller from [8] to ensure that the desired formation is acquired and maneuvers according to a given translational velocity. We then present an experimental evaluation of the proposed control scheme using the wheeled robot platform GRITSBot [38]. The results show the successful implementation of the formation control algorithm.

II. Background Material

An undirected graph G is a pair (V, E) where $V = \{1, 2, ..., n\}$ is the set of nodes and $E \subset V \times V$ is the set of undirected edges that connect two different nodes, i.e., if node pair $(i, j) \in E$ then so is (j, i). We let $a \in \{1, \ldots, N(N-1)/2\}$ denote the total number of edges in E. The set of neighbors of node i is denoted by

$$\mathcal{N}_i(E) = \{j \in V \mid (i, j) \in E\}. \quad (1)$$

If $p_i \in \mathbb{R}^2$ is the coordinate of node i, then a framework F is defined as the pair (G, p) where $p = [p_1, \ldots, p_2] \in \mathbb{R}^{2n}$. In the following, we assume all frameworks have *generic* properties, i.e., the properties hold for almost all of the framework representations. This is done to exclude certain degenerate configurations such as frameworks that lie in a hyperplane (see [21] for a detailed study of generic framework).

Based on an arbitrary ordering of edges, the edge function $\phi : \mathbb{R}^{2n} \to \mathbb{R}^a$ is given by

$$\phi(p) = \left[..., \|p_i - p_j\|^2, ...\right], \quad (i, j) \in E \quad (2)$$

such that its kth component, $\|p_i - p_j\|^2$, relates to the kth edge of E connecting the ith and jth nodes. The rigidity matrix $R : \mathbb{R}^{2n} \to \mathbb{R}^{a \times 2n}$ is given by

$$R(p) = \frac{1}{2} \frac{\partial \phi(p)}{\partial p} \quad (3)$$

where $\text{rank}[R(p)] \leq 2n - 3$ [2].

An isometry of $\mathbb{R}^2$ is a bijective map $\mathcal{T} : \mathbb{R}^2 \to \mathbb{R}^2$ satisfying [23]

$$\|w - z\| = \|\mathcal{T}(w) - \mathcal{T}(z)\|, \quad \forall w, z \in \mathbb{R}^2. \quad (4)$$

This map includes rotations and translations of the vector $w - z$. Two frameworks are said to be *isomorphic* in $\mathbb{R}^2$ if they are related by an isometry. In this paper, we will represent the collection of all frameworks that are isomorphic to F by $\text{Iso}(F)$. It is important to point out that (2) is invariant under isomorphic motions of the framework.

Frameworks (G, p) and $(G, \hat{p})$ are equivalent if $\phi(p) = \phi(\hat{p})$, and are congruent if $\|p_i - p_j\| = \|\hat{p}_i - \hat{p}_j\|, \forall i, j \in V$ [24]. The necessary and sufficient condition for a generic framework (G, p) to be infinitesimally rigid is $\text{rank}[R(p)] = 2n - 3$ [23]. An infinitesimally rigid framework is minimally rigid if and only if $a = 2n - 3$ [1]. If the infinitesimally rigid frameworks (G, p) and $(G, \hat{p})$ are equivalent but not congruent, then they are referred to as *ambiguous* [1]. The notation $\text{Amb}(F)$ will be used here to represent the collection of all frameworks that are ambiguous to the infinitesimally rigid framework F. All frameworks in $\text{Amb}(F)$ are also assumed to be infinitesimally rigid. According to [1] and Theorem 3 of [3], this assumption holds almost everywhere; therefore, it is not restrictive.

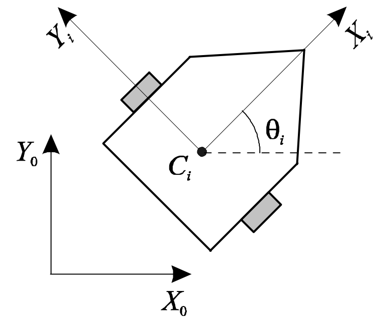

Fig. 1. Top view of the unicycle agent.

Lemma 1: [10] Consider two frameworks $F = (G, p)$ and $\bar{F} = (G, \bar{p})$ sharing the same graph $G = (V, E)$ and the function

$$\Lambda(\bar{F}, F) = \sum_{(i,j) \in E} \left(\|\bar{p}_i - \bar{p}_j\| - \|p_i - p_j\|\right)^2. \quad (5)$$

If F is infinitesimally rigid and $\Lambda(\bar{F}, F) \leq \varepsilon$ where ε is a sufficiently small positive constant, then $\bar{F}$ is also infinitesimally rigid.

Lemma 2: [8] For any $x \in \mathbb{R}^2$, $R(p)(1_n \otimes x) = 0$ where 1_n is the $n \times 1$ vector of ones.

Finally, the following metric will be used to denote the "distance" between a point and a set:

$$\text{dist}(\zeta, \mathcal{M}) = \inf_{x \in \mathcal{M}} \|\zeta - x\| \quad (6)$$

for points $\zeta, x \in \mathbb{R}^2$ and a set $\mathcal{M}$.

III. System Model

Consider a system of n agents moving autonomously on the plane. Figure 1 depicts the ith agent, where the reference frame $\{X_0, Y_0\}$ is fixed to the Earth. The moving reference frame $\{X_i, Y_i\}$ is attached to the ith vehicle with the X_i axis aligned with its heading (longitudinal) direction, which is given by angle θ_i and measured counterclockwise from the X_0 axis. Point C_i denotes the ith vehicle's center of mass which is assumed to coincide with its center of rotation.

We assume the agent motion is governed by the following nonholonomic, unicycle kinematic model

$$\dot{q}_i = S(\theta_i)\eta_i, \quad i = 1, ..., n. \quad (7)$$

In (7), $q_i = [x_i, y_i, \theta_i]$ denotes the position and orientation of $\{X_i, Y_i\}$ relative to $\{X_0, Y_0\}$, $\eta_i = [v_i, \omega_i]$ is the control input, v_i is the ith agent's translational speed in the direction of θ_i, ω_i is the ith agent's angular speed about the vertical axis passing through C_i, and

$$S(\theta_i) = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix}. \tag{8}$$

IV. Problem Statement

Consider that the agents' target formation is modeled by the framework $F^* = (G^*, p^*)$ where $G^* = (V^*, E^*)$, $\dim(V^*) = n$, $\dim(E^*) = a$, $p^* = [p_1^*, \ldots, p_n^*]$, and $p_i^* = [x_i^*, y_i^*]$. The fixed target distance separating the ith and jth agents is given by

$$d_{ij} = \left\| p_i^* - p_j^* \right\| > 0, \quad i, j \in V^*. \tag{9}$$

We assume F^* is constructed to be infinitesimally and minimally rigid. The actual formation of the agents is encoded by the framework $F(t) = (G^*, p(t))$ where $p = [p_1, \ldots, p_n]$ and $p_i = [x_i, y_i]$.

The statement of our control problem is the following.

Translational Maneuvering Problem: The agents need to acquire and maintain a pre-defined geometric shape in the plane while simultaneously moving with a given velocity. That is,

$$F(t) \to \text{Iso}\,(F^*) \text{ as } t \to \infty, \tag{10}$$

which is equivalent to

$$\|p_i(t) - p_j(t)\| \to d_{ij} \text{ as } t \to \infty, \quad i, j \in V^* \tag{11}$$

due to the framework rigidity, and

$$\dot{p}_i(t) - v_t(t) \to 0 \text{ as } t \to \infty, \quad i = 1, \ldots, n \tag{12}$$

where $v_t \in \mathbb{R}^2$ is any continuously differentiable function of time representing the desired translational velocity. We assume v_t and $\dot{v}_t$ are bounded for all time.

V. Translational Maneuvering Control

Before presenting the control scheme, we introduce several error variables.

The relative position of agents i and j is defined as

$$\tilde{p}_{ij} = p_i - p_j, \tag{13}$$

while their distance error is captured by the variable [29]

$$z_{ij} = \|\tilde{p}_{ij}\|^2 - d_{ij}^2. \tag{14}$$

Given that $\|\tilde{p}_{ij}\| \geq 0$, note that $z_{ij} = 0$ if and only if $\|\tilde{p}_{ij}\| = d_{ij}$. Finally, let

$$\tilde{\theta}_i = \theta_i - \theta_{di} \tag{15}$$

where θ_{di} denotes the desired heading direction, which is to be specified later.

Theorem 1: Let the initial conditions for the distance errors be $z(0) \in \Omega_1 \cap \Omega_2$

$$\Omega_1 = \{z \in \mathbb{R}^a \mid \Lambda(F, F^*) \leq \delta\}$$

$$\Omega_2 = \{z \in \mathbb{R}^a \mid \text{dist}(p, \text{Iso}(F)) < \text{dist}(p, \text{Amb}(F^*))\}, \tag{16}$$

$z = [\ldots, z_{ij}, \ldots] \in \mathbb{R}^a$, $(i,j) \in E^*$ is ordered as (2), and δ is a sufficiently small positive constant. Then, the kinematic control law

$$v_i = u_{ix}\cos\theta_i + u_{iy}\sin\theta_i \tag{17a}$$

$$\omega_i = -\beta_i\tilde{\theta}_i + \dot{\theta}_{di}, \quad i \in V_d \tag{17b}$$

$$u_i = \begin{bmatrix} u_{ix} \\ u_{iy} \end{bmatrix} = -k \sum_{j \in \mathcal{N}_i(E^*)} \tilde{p}_{ij} z_{ij} + v_t \tag{17c}$$

$$\theta_{di} = \begin{cases} 0, & \text{if } u_{ix} = u_{iy} = 0 \\ \text{atan2}(u_{iy}, u_{ix}), & \text{otherwise,} \end{cases} \tag{17d}$$

where β_i and k are positive constant control gains, ensures $(z, \tilde{\theta}_i) = 0$ for all $i \in V^*$ is exponentially stable and that (10) and (12) hold.

Proof: We first decompose (7) as follows

$$\dot{p}_i = \begin{bmatrix} v_i\cos\theta_i \\ v_i\sin\theta_i \end{bmatrix} \tag{18}$$

$$\dot{\theta}_i = \omega_i. \tag{19}$$

Using (17a) and (17d) in (18), one can see that

$$\dot{p}_i = u_i. \tag{20}$$

Now, consider the Lyapunov function candidate

$$V = \frac{1}{4} \sum_{(i,j) \in E^*} z_{ij}^2 = \frac{1}{4} z^\intercal z. \tag{21}$$

Since

$$\dot{z}_{ij} = \frac{d}{dt}\left(\tilde{p}_{ij}^\intercal \tilde{p}_{ij}\right) = 2\tilde{p}_{ij}^\intercal (u_i - u_j) \tag{22}$$

from (14), the derivative of (21) is given by

$$\dot{V} = \sum_{(i,j) \in E^*} z_{ij}\tilde{p}_{ij}^\intercal (u_i - u_j) = z^\intercal R(p) u \tag{23}$$

where (3) was used and $u = [u_1, \ldots, u_n] \in \mathbb{R}^{2n}$.

From (17c), we have that

$$u = -kR^\intercal(p) z + 1_n \otimes v_t. \tag{24}$$

Substituting (24) into (23) yields

$$\dot{V} = -kz^\intercal R(p) R^\intercal(p) z \tag{25}$$

where Lemma 2 was used. Given that F^* and $F(t)$ have the same edge set and F^* is minimally rigid by design, then $F(t)$ is minimally rigid for all $t \geq 0$. Moreover, from Lemma 1 and the fact that F^* is infinitesimally rigid, we know $F(t)$ is infinitesimally rigid for $z(t) \in \Omega_1$. Therefore, $R(p)$ has full row rank and

$$\dot{V} \leq -k\underline{\lambda} z^\intercal z \quad \text{for} \quad z(t) \in \Omega_1 \tag{26}$$

where $\underline{\lambda} = \inf_t \lambda_{\min}(RR^\intercal) > 0$ and $\lambda_{\min}$ represents the minimum eigenvalue. From (26), we know that $\dot{V}(t) \leq 0$ for all $t \geq 0$, which implies that $V(t) \leq V(0)$ for all $t > 0$. Therefore, since $z(t) \in \Omega_1$ is equivalent to $z(t) \in \left\{z \in \mathbb{R}^{2n} \mid V(z) \leq c\right\}$ according to Lemma 2 of [9], a sufficient condition for (26) is given by

$$\dot{V} \leq -4k\underline{\lambda} V \quad \text{for} \quad z(0) \in \Omega_1. \tag{27}$$

From (27), we know that $z = 0$ is exponentially stable for $z(0) \in \Omega_1 \cap \Omega_2$ [26], and therefore (11) is met.

The exponential stability of $z = 0$ infers one of two possible occurrences: $F(t) \to \text{Iso}(F^*)$ or $F(t) \to \text{Amb}(F^*)$ as $t \to \infty$. Since the initial condition is such that $z(0) \in \Omega_1 \cap \Omega_2$, then we have from (16) that

$$\text{dist}(q(0), \text{Iso}(F^*(0))) < \text{dist}(q(0), \text{Amb}(F^*(0))). \quad (28)$$

It follows from this condition that the energy function (21) would necessarily have to increase for a certain time interval for $F(t) \to \text{Amb}(F^*)$ as $t \to \infty$ to occur. This is however contradictory to the fact that $V(t)$ is nonincreasing for all time. Thus, we conclude that $F(t) \to \text{Iso}(F^*)$ as $t \to \infty$ for $z(0) \in \Omega_1 \cap \Omega_2$.

Since $z(t)$ is bounded, we know from (14) that $\tilde{p}_{ij}(t)$, $(i,j) \in E^*$ is bounded. Therefore, since $z(t) \to 0$ as $t \to \infty$, we know from (17c) and (20) that $\dot{p}_i(t) - v_t(t) \to 0$ as $t \to \infty$ for $\forall i \in V^*$.

Finally, after taking the derivative of (15) and substituting (19) and (17b), we obtain

$$\dot{\tilde{\theta}}_i = -\beta_i \tilde{\theta}_i, \quad (29)$$

which indicates that $\tilde{\theta}_i = 0, \forall i \in V^*$ is exponentially stable. ■

Remark 1: The time derivative of (17d), which is needed in (17b), can be calculated as follows

$$\dot{\theta}_{di} = \begin{cases} 0, & \text{if } u_{ix} = u_{iy} = 0 \\ \frac{-u_{iy}}{u_{ix}^2 + u_{iy}^2}\dot{u}_{ix} + \frac{u_{ix}}{u_{ix}^2 + u_{iy}^2}\dot{u}_{iy}, & \text{otherwise} \end{cases} \quad (30)$$

where

$$\dot{u}_i = -k \sum_{j \in \mathcal{N}_i(E^*)} \left(z_{ij} + 2\tilde{p}_{ij}\tilde{p}_{ij}^\intercal\right)(u_i - u_j) + \dot{v}_t \quad (31)$$

and (20) and (22) were used.

Remark 2: The proposed control scheme assumes that the time-varying velocity $v_t(t)$ is known to all agents. This is not an overly restrictive assumption since in many cases this information is known beforehand and can pre-programmed into the agents' onboard computer. Note that for the case where v_t is constant and only available to a subset of agents, one can use a distributed observer to estimate the desired flocking velocity by exploiting the connectedness of the formation graph [34].

Remark 3: The control (17) is time invariant and discontinuous, which is expected by Brockett's condition for stabilization of nonholonomic systems [6]. Interestingly, the experimental study in [28] has shown that such controllers can yield better performance than time-varying, continuous controls if carefully implemented on unicycle-type robots.

VI. Experimental Results

To demonstrate the performance of the kinematic controller from Section V, we conducted an experiment on the *Robotarium* system [39], which is a swarm robotics testbed located at the Georgia Institute of Technology that uses the GRITSBot as the mobile robot platform [38]. The testbed arena has a 8×12 ft^2 area on which multiple robots can be deployed. The GRITSBot is a low-cost, wheeled robot equipped with a suite of onboard sensors, wireless communication, battery, and processing boards, and has a footprint of approximately 3×3 cm^2. An overhead camera and a unique identification tag atop each robot's chassis provide a position tracking system for their motion. A picture of Robotarium and the GRITSBot are shown in Figure 2. Robotarium is ideal for testing kinematic control laws since it uses velocity-level commands as inputs to the robots with the low-level, velocity control loop being invisible to the user.

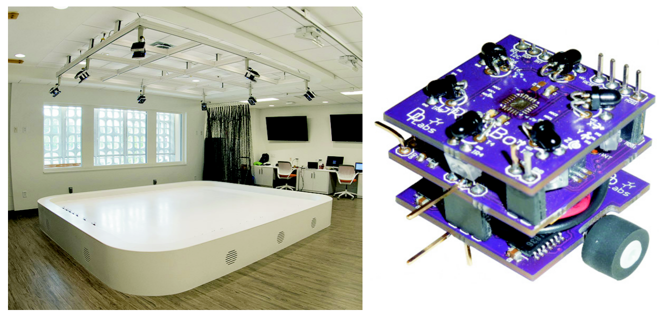

Fig. 2. The Robotarium (left) and the GRITSBot (right).

The experiment was conducted with five robots. The desired formation F^* was set to a regular pentagon, which was made infinitesimally and minimally rigid by introducing seven edges such that $E^* = \{(1,2),(1,3),(1,4),(1,5),(2,3),(3,4),(4,5)\}$. The desired distances between all robots were given by $d_{12} = d_{23} = d_{34} = d_{45} = d_{15} = \alpha\sqrt{2(1-c_1)}$ and $d_{13} = d_{14} = \alpha\sqrt{2(1+c_2)}$ where $\alpha = 0.1$ m, $s_1 = \sin\frac{2\pi}{5}$, $s_2 = \sin\frac{4\pi}{5}$, $c_1 = \cos\frac{2\pi}{5}$, and $c_2 = \cos\frac{\pi}{5}$. The formation was required to move as a virtual rigid body around a circle. To this end, the desired translational maneuvering velocity was chosen as $v_t(t) = [-rb\sin bt, rb\cos bt]$ m/s where $r = 0.15$ m is the radius for the circular trajectory and $b = 0.3$ rad/s. Figure 3 depicts the desired formation and desired maneuver. The initial positions and orientations of the robots were randomly selected. The control gains in (17b) and (17c) were set to $\beta_i = 10$, $i = 1, ..., 5$, and $k = 6$.

Snapshots of the formation at $t = 0$ s and $t = 32$ s are given in Figure 4 showing that the desired formation was successfully acquired from the random initial configuration. The path of the geometric center of the formation as it maneuver in the circle is shown in Figure 5. Figure 6 shows the inter-agent distance errors and heading angle errors quickly converging to approximately zero. The errors are not exactly zero due to measurement noise and the sensor resolution. We can observe from the errors that the desired formation is acquired after approximately 25 s. The control inputs are depicted in Figure 7, where one can see that $v_i(t) \to rb = 4.5$ cm/s as $t \to \infty$ for all i. The steady-state values for $\omega_i(t)$, $i = 1, ..., 5$, are approximately 0.5 rad/s rather than the expected value of $b = 0.3$ rad/s. This can be explained from (17b) by the facts that a) the term

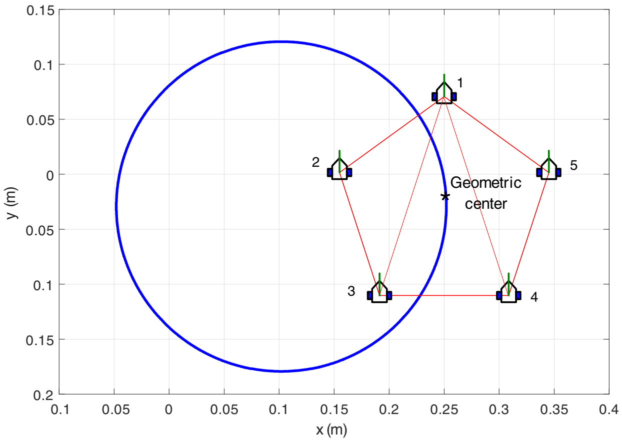

Fig. 3. Desired pentagon formation along with desired circular trajectory for the geometric center.

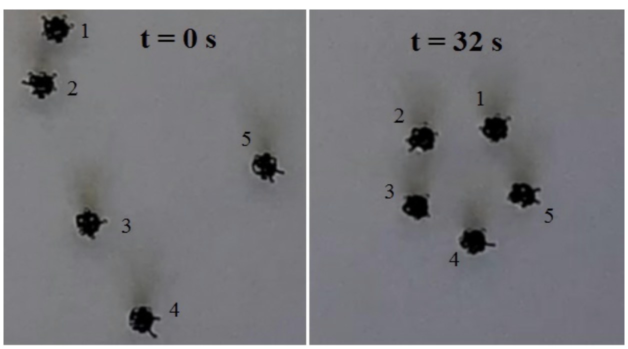

Fig. 4. Snapshots of the initial formation (left) and the formation when the desired formation was acquired (right).

$\beta_i\tilde{\theta}_i(t)$ does not approach zero as $t \to \infty$ due to the small steady-state offset and fluctuation in $\tilde{\theta}_i$, and b) the term $\dot{\theta}_{di}(t)$ does not approach 0.3 as $t \to \infty$ due to the manner in which (30) is calculated. Also, notice from Figure 7 that the Robotarium testbed limits the robot's linear velocity to ± 10 cm/s. A video of the experiment can be seen in https://youtu.be/2EV_SUpvsrk.

VII. Conclusion

This paper demonstrated how the distance-based, formation maneuvering controller, originally designed for single-integrator models, can be applied to nonholonomic kinematic agents. An input transformation along with a Lyapunov analysis showed that the proposed control ensures exponential convergence to the desired formation while maneuvering according to the desired translational velocity. An experimental validation of the formation control scheme was presented using unicycle-type robotic vehicles.

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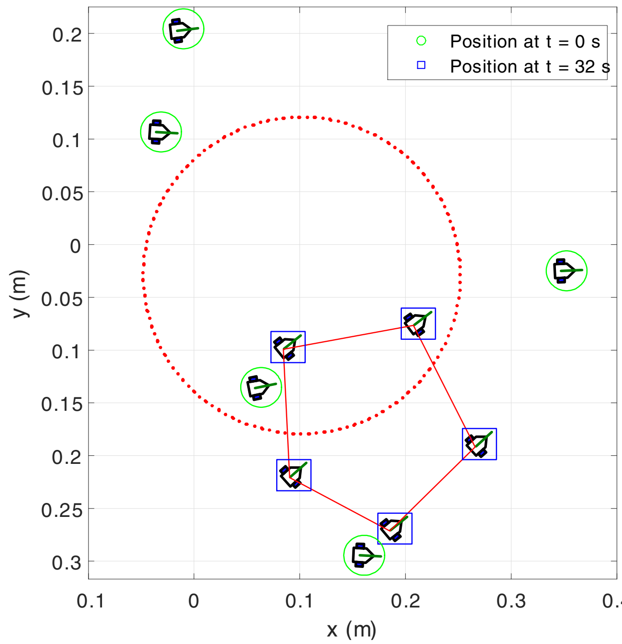

Fig. 5. Circular maneuver of the robots from the initial formation.

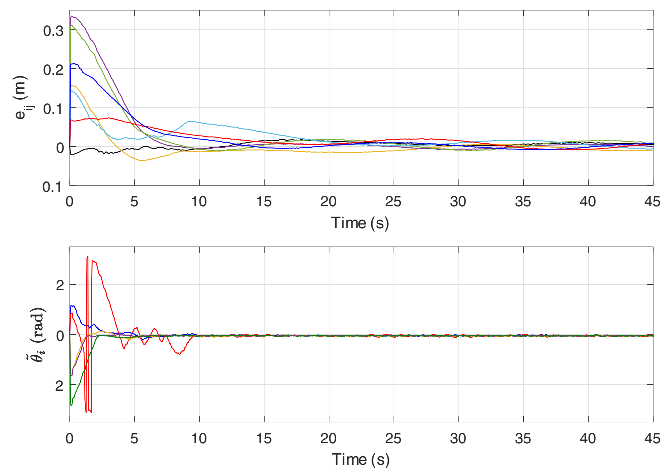

Fig. 6. Distance errors, $e_{ij}(t)$, $i, j \in V^*$ (top) and heading angle errors, $\tilde{\theta}_i(t)$, $i = 1, ..., 5$ (bottom).

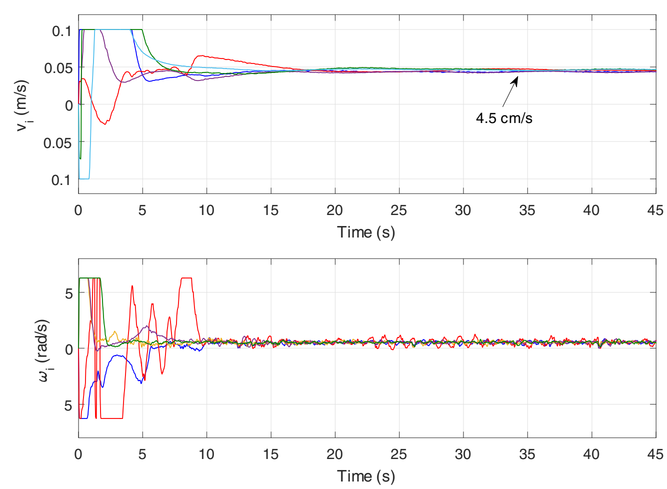

Fig. 7. Control inputs $\eta_i(t) = [v_i(t), \omega_i(t)]$, $i = 1, \ldots, 5$.

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