2014 American Control Conference (ACC)
June 4-6, 2014. Portland, Oregon, USA

Distance-based Formation Control with a Single Moving Leader

Sung-Mo Kang[1], Myoung-Chul Park[1], Byung-Hun Lee[1], and Hyo-Sung Ahn[1]

Abstract— **This paper proposes distance-based adaptive formation control laws for the leader-follower system. The developed controller makes all the agents maintain the formation group and move with a constant reference velocity in a plane. It is assumed that there are one leading and two following agents. The leading agent knows the reference velocity whereas the follower does not know the velocity of other agents. Thus, to move in a group, the controller for the follower estimates the reference velocity. An adaptive method is used in the estimation process. The stability and boundedness of the formation are proved by using Lyapunov stability analysis and Barbalat's lemma. Simulations results are included to illustrate the validity of the developed theories.**


(a) The formation of birds


(b) The formation of planes

Fig. 1: Examples of formation in nature

I. INTRODUCTION

In our lives, we see many formations such as flocks of birds or clusters of planes as Fig. 1. Animals in nature form a flock or a school because formation behaviors provide benefits to the animals. For instance, an animal in a herd has small chance to be directly attacked from predators. By grouping, animals also can integrate sense to maximize the chance of detecting predators or foraging for food. Research on the flock and school show that these behaviors are combination of staying in the group and keeping a distance from other members of the group. That is, the animals form a specific formation by preserving the distance between each other. Groups of artificial agents can get the similar benefits. In formation, the range of detection can be large, and the responsibility of each agent can be reduced. They can accomplish a mission efficiently. Therefore, it is useful and valuable to keep the formation. As a result, in recent years, the research on the formation control of unmanned multi-agent systems has received significant attention [1], [2], [3], [4], [5], [6]. In particular, an interest in the distance-based formation control has been increasing gradually.

The formation control problems are categorized into several parts according to the information architecture utilized; for example, position-based, displacement-based, and distance-based formation control problems have been studied [2]. The latter two architectures are more challenging relative to the first one since they use only local information for formation control. In the displacement-based control, agents measure the relative positions of their neighbors with respect to a global reference frame. In this case, the orientations of local reference frame of each agent are aligned to a global reference frame. Thus, the displacement of each agent can be directly controlled to achieve its desired formation (e.g. [4], [5], [6]). On the other hand, for the case of the distance-based control, agents measure the relative positions of their neighbors with respect to their own local reference frames that are not aligned with each other. Therefore, in the distance-based formation control, displacement of each agent cannot be directly controlled; instead, agents use distances between each other as a control variable to achieve and maintain the desired formation (e.g. [1], [3], [7], [8], [9], [10]).

Since available information is restricted in distance-based control, it makes the problem quite complicated; thus, it is not easy to solve it in general. However, there are several reasons why pursuing a research on the distance-based formation control is extremely useful and valuable. That is, in the case of distance-based formation control, the controllers for each agent, except the leading agent, only need the relative positions of neighbors with respect to their own local reference frames. This means that each agent requires less equipment and the controllers for each agent are perfectly decentralized. Thus, a global sensing is not required in the distance-based setups, which implies that the distance-based approaches have better cost-effectiveness, scalability, and robustness than the position-based and displacement-based approaches.

Most of the previous research on the distance-based formation control has considered only the shape of the formation (e.g. [1], [7], [8], [11]). However, in practical situations such as unmanned aerial vehicles or airplanes, the agents should not only maintain the shape of formation but also move with a given reference velocity. There are a number of research works that consider movement of formation in the displacement-based control, but for the distance-based control, only few results are available because of its extremely complexity in analysis. The movement of formation was handled recently only in several articles such as [3], [9], [10], [12], [13]. The convergence of formation, when the

[1]School of Mechatronics, Gwangju Institute of Science and Technology (GIST), 261 Cheomdan-gwagiro, Buk-gu, Gwangju, 500-712 Korea. {smkang0521, mcpark, bhlee, hyosung}@gist.ac.kr

978-1-4799-3274-0/$31.00 ©2014 AACC

reference velocity is constant, was mentioned in [3]; however unfortunately, there is a wrong proof of theorem. A moving formation with two leaders was studied in [13].

In this paper, a distance-based formation control law is proposed, where both the shape and movement of formation are considered. Contrary to [13], this paper particularly focuses on the case where there are one leading agent and two following agents. All of the agents measure only relative positions of their neighbors with respect to their own local reference frames. The difference between the leader and followers is that the leader knows the reference velocity and the followers do not know it. Therefore, the follower is supposed to estimate the reference velocity. To solve this additional issue, using an adaptive control method, the follower estimates the reference velocity. After that, using this adaptively estimated velocity, the desired formation is achieved only using relative distance information among agents.

The outline of this paper is as follows. In Section II, we introduce background and preliminaries of this research and, in Section III, the problem studied in this paper is explicitly defined. The control law for the leader-follower formation and stability analysis are presented in Section IV. The simulation results are shown in Section V and the conclusion and future works are described in Section VI.

II. BACKGROUND AND PRELIMINARIES

A. Graph

Directed graph is represented by a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, ..., N\}$ is the set of vertices and $\mathcal{E} = \{..., (i,j), ...\} \subset \mathcal{V} \times \mathcal{V}$ is the set of directed edges. The N is the number of vertices, and an edge (i,j) is considered to be directed from j to i. The set of neighbors of $i \in \mathcal{V}$ is defined as $\mathcal{N}_i = \{j \in \mathcal{V} | \, (i,j) \in \mathcal{E}\}$. The pair $(\mathcal{G}, x)$ is called a framework, where $x_i \in \mathbb{R}^2$ is position of vertex i and $x = \left[x_1^T \cdots x_N^T\right]^T \in \mathbb{R}^{2N}$ is a realization of $\mathcal{G}$. The position of vertex i is fitting if there is no position $x_i^* \in \mathbb{R}^2$ for i such that

$$\{(i,j) \in \mathcal{E} | \, \|x_i - x_j\| = d_{ij}\} \subset \{(i,j) \in \mathcal{E} | \, \|x_i^* - x_j\| = d_{ij}\}$$

where $d_{ij} > 0$ is the desired distance.

In this paper, the desired formation is supposed to be persistent. The definition of persistent is given as follows [14]:

Definition 2.1: A representation x is persistent if there exists $\varepsilon > 0$ such that every representation x' is fitting for the distance set induced by x and satisfying $d(x, x') = \max_{i \in \mathcal{V}} \|x_i - x_i{}'\| < \varepsilon$ is congruent to x. A graph is generically persistent if almost all its representations are persistent.

B. Input-to-state stability

Consider a system

$$\dot{x} = f(x, u) \tag{1}$$

where $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is continuous in time and Lipschitz in x and u. The input $u(t)$ is continuous and bounded function of t for all $t \geq 0$. Given the system (1), input-to-state stability is defined as follows:

Definition 2.2 ([15]): The system (1) is said to be input-to-state stable if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that for any initial state $x(t_0)$ and any bounded input $u(t)$, the solution $x(t)$ exists for all $t \geq t_0$ and satisfies

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0) + \gamma\left(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\|\right)$$

The next theorem presents a sufficient condition for input-to-state stability.

Theorem 2.1 ([15]): Suppose $f(x, u)$ is continuous differentiable and Lipschitz in (x, u), uniformly in t. If the unforced system $\dot{x} = (x, 0)$ has a exponentially stable equilibrium point at the origin $x = 0$, then the system (1) is input-to-state stable.

C. Notation

In this paper, the following notation is used, $\Theta(\cdot)$, which is defined as follows:

Definition 2.3 (Big-theta, $\Theta(\cdot)$): Let $f(n)$ and $g(n)$ be functions defined on some subset of the real numbers. We say that $f(n)$ is $\Theta(g(n))$ (or $f(n) \in \Theta(g(n))$) if there exist real numbers $k_1 > 0$ and $k_2 > 0$ and there exists a real number n_0 such that $k_1 g(n) \leq f(n) \leq k_2 g(n)$ for every real number $n \geq n_0$.

III. PROBLEM STATEMENT

In this paper, it is supposed that the motion model of each agent is a single integrator. Therefore, the system in two dimension is expressed as

$$\dot{p}_i = u_i, \qquad i = 1, ..., N \tag{2}$$

where $p_i \in \mathbb{R}^2$, $u_i \in \mathbb{R}^2$ and $N \in \mathbb{R}$ are the position, the control input for agent i and the number of agents, respectively. This paper considers only three agents; thus N is three.

The first and the second agents are called respectively leader and first follower. The other agents are called followers. Each agent measures the relative position of a neighboring agent $j \in \mathcal{N}_i$ with respect to a local reference frame. Because the leader has no neighbor, it is not affected by the other agents. However, the other agents including the first follower are affected by the other agents because they have one or more neighbors.

Fig. 2 shows the direction of the interactions between agents. The arrow from agent j to agent i means that the agent j measures the relative position of agent i and uses this information for positioning.

$z_{ij} \in \mathbb{R}^2$ and $e_{ij} \in \mathbb{R}$ are used to represent the relative position vector and the distance error:

$$z_{ij} = p_i - p_j, \qquad \forall (i,j) \in \mathcal{E} \tag{3}$$

$$e_{ij} = \|z_{ij}\|^2 - d_{ij}^2, \quad \forall (i,j) \in \mathcal{E} \tag{4}$$

where $d_{ij} \in \mathbb{R}$ is the desired distance between agent i and j.

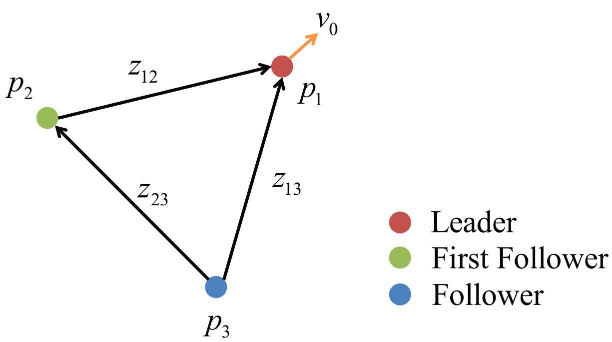

Fig. 2: Interactions in the directed graph when the number of agents are three.

The several conditions of system are assumed.

Assumption 3.1: The desired formation is minimally persistent and has three agents called leader, first follower and follower (i.e., leader-follower system).

Assumption 3.2: The leading agent moves with reference velocity $v_0 \in \mathbb{R}^2$ which is constant. However, the following agents including the first follower do not know the velocity of leading agent or other agent.

In this paper, the distance-based formation control law for the system (2) is designed and the stability of system is analyzed.

IV. STABILITY ANALYSIS

In the Assumption 3.2, it is assumed that the leader moves with reference velocity v_0. Therefore, the dynamics of leader is represented as:

$$\dot{p}_1 = v_0$$

A. Stability of the first follower

As mentioned Section III, the first follower measures only the relative position of leading agent and have to keep the desired distance. In the previous research, the distance error of first follower is not converged to zero when the leading agent is moving. The only boundedness of error is proved when the leading agent is moving with reference velocity.

In this paper, the adaptive method is considered to estimate the velocity of leading agent and to keep the desired distance. The control law and the estimator for the first follower are proposed as:

$$u_2 = \hat{v}_2 + k_0 z_{12} e_{12} \quad (5)$$

$$\dot{\hat{v}}_2 = z_{12} e_{12} \quad (6)$$

where k_0 is an arbitrary positive constant. The velocity of convergence is changed according to the k_0.

Consider the Lyapunov candidate to analyze stability of the first follower:

$$V_1 = \frac{1}{2} e_{12}^2 + \|v_0 - \hat{v}_2\|^2 \quad (7)$$

which is continuously differentiable.

The time derivative of V_1 is

$$\dot{V}_1 = -2k_0 e_{12}^2 \|z_{12}\|^2 \leq 0 \quad (8)$$

From (7) and (8), it is clear that the derivative of Laypunov candidate is positive semi-definite. Therefore, we can obtain the following result.

Theorem 4.1: The distance error e_{12}^2 and the adaptation error $\|v_0 - \hat{v}_2\|^2$ are bounded and smaller than initial values.

The next result is obtained also from (7) and (8).

Lemma 4.1: The state of system converges to one of the following two cases:

1) The distance error converges to zero (i.e., $e_{12} \to 0$).
2) The position of first follower becomes coincident with leading agent (i.e., $z_{12} \to 0$).

Proof: The Lyapunov candidate was defined as (7). Therefore, V_1 is lower bounded. From (8), the time derivative of Lyapunov candidate is negative semi-definite. Further, $\dot{V}_1$ is uniformly continuous because e_{12}^2 and $\|v_0 - \hat{v}_2\|^2$ are bounded. These conditions satisfy Barbalat's lemma [15] and imply $\dot{V}_1 \to 0$ (i.e., $e_{12}^2 \|z_{12}\|^2 \to 0$) as $t \to \infty$. Because e_{12} is defined as (4), both e_{12}^2 and $\|v_0 - \hat{v}_2\|^2$ cannot be zero at the same time. As a result, the state of system converges to one of two cases; $e_{12} \to 0$ or $z_{12} \to 0$. ■

In Theorem 4.1 and Lemma 4.1, the boundedness of e_{12}^2 and $\|v_0 - \hat{v}_2\|^2$ is proved and the convergence of $e_{12}^2 \|z_{12}\|^2$ is also shown. By using these results, the convergence of estimate error is obtained.

Theorem 4.2: The estimator (6) can estimate the velocity of leading agent. That is, the estimate error converges to zero (i.e., $(v_0 - \hat{v}_2) \to 0$ as $t \to \infty$).

Proof: Because both e_{12} and $v_0 - \hat{v}_2$ are bounded as proved in Theorem 4.1, $\dot{\hat{v}}_2 = z_{12} e_{12}$ is uniformly continuous and $\hat{v}_2$ is also bounded. Further, $\dot{\hat{v}}_2$ converges to zero because $e_{12}^2 \|z_{12}\|^2$ converge to zero as shown in Lemma 4.1. Therefore, it is concluded that $\hat{v}_2$ converges to arbitrary constant velocity.

Consider

$$\begin{aligned} z_{12} &= p_1 - p_2 = \int (\dot{p}_1 - \dot{p}_2)\, dt + p_1(0) + p_2(0) \\ &= \int (v_0 - \hat{v}_2 - k_0 e_{12} z_{12})\, dt + p_1(0) + p_2(0) \end{aligned}$$

Therefore,

$$\begin{aligned} \int (v_0 - \hat{v}_2)\, dt &= z_{12} + k_0 \int e_{12} z_{12} dt - p_1(0) - p_2(0) \\ &= z_{12} + k_0 \hat{v}_2 - p_1(0) - p_2(0) \end{aligned}$$

It is already known that z_{12} and $\hat{v}_2$ converge to arbitrary constant. Therefore, $v_0 - \hat{v}_2$ is converge to zero by using Barbalat's lemma. ■

Next, the convergence of distance error is proved by using the input-to-state stability. Therefore, the input-to-state stable of distance error has to be shown at first.

Consider the distance error dynamics as follows:

$$\begin{aligned} \dot{e}_{12} &= 2z_{12}^T \dot{z}_{12} = 2z_{12}^T (v_0 - \hat{v}_2 - k_0 z_{12} e_{12}) \\ &:= f_1(e_{12}, v_0 - \hat{v}_2) \end{aligned} \quad (9)$$

where f_1 with $v_0 - \hat{v}_2$ as input is continuously differentiable and Lipschitz.

Theorem 4.3: The distance error dynamics (9) with $v_0 - \hat{v}_2$ as input is input-to-state stable unless the leading agent and the first follower are initially coincident (i.e., $z_{12}(0) \neq 0$).

Proof: From the result in Theorem 4.1, it is easily obtained that f_1 is continuously differentiable and Lipschitz. The unforced system of (9) is as belows:

$$\begin{aligned} \dot{e}_{12} &= f_1(e_{12}, 0) \\ &= 2z_{12}^T(-k_0 z_{12} e_{12}) = -2k_0 \|z_{12}\|^2 e_{12} \end{aligned}$$

Define a Lyapunov candidate to analyze the stability of unforced system:

$$c_1 e_{12}^2 \leq V_2 = \frac{1}{2} e_{12}^2 \leq c_2 e_{12}^2$$

where c_1 and c_2 are positive constants and satisfy $c_1 \leq \frac{1}{2}$ and $c_1 \geq \frac{1}{2}$. Therefore, the time derivative of Lyapunov candidate is

$$\dot{V}_2 = e_{12}\dot{e}_{12} = -2k_0 \|z_{12}\|^2 e_{12}^2 \leq -c_3 e_{12}^2$$

where c_3 is positive and satisfies $c_3 \leq 2k_0\|z_{12}\|^2$. As a result, the unforced system is exponentially stable unless $\|z_{12}\|$ is initially zero. These results imply that f_1 is input-to-state stable by using Theorem 2.1. ■

By using Theorem 4.2 and Theorem 4.3, it is concluded that the first follower converge to the desired formation.

Theorem 4.4: In the system (2), unless the agents are initially coincident(i.e., $p_1(0) \neq p_2(0)$), the control law (5) and the estimator (6) make the first follower converge to the desired formation with the distance given as d_{12}.

Proof: In Theorem 4.3, it was proved that the distance error dynamics (9) with $v_0 - \hat{v}_2$ as input is input-to-state stable. Further, in Theorem 4.2, it was shown that $v_0 - \hat{v}_2$ converges asymptotically to zero. Because the input of distance error dynamics converges to zero, the distance error e_{12} converges asymptotically to zero as $t \to \infty$. That is, $\|z_{12}\| \to d_{12}$ as $t \to \infty$.

The convergence of distance error implies that $\dot{p}_2 \to \hat{v}_2$ in (2) and (5). Therefore, the velocity of first follower converges to the velocity of leading agent (i.e., $\dot{p}_2 \to v_0$ as $t \to \infty$). ■

B. Stability of the follower

The follower measures the relative positions of neighbors such as the leading agent and the first follower. Using these informations, the control law and the estimator for the follower are proposed as:

$$\begin{aligned} u_3 &= \hat{v}_3 + z_{13}e_{13} + z_{23}e_{23} && (10) \\ \dot{\hat{v}}_3 &= z_{13}e_{13} + z_{23}e_{23} && (11) \end{aligned}$$

To investigate boundedness of the follower, consider a following function:

$$V_3 = \frac{1}{4}\|e_{13}\|^2 + \frac{1}{4}\|e_{23}\|^2 + \frac{1}{2}\|v_0 - \hat{v}_3\|^2 \quad (12)$$

Then,

$$\begin{aligned} \dot{V}_3 &= -\|e_{13}z_{13} + e_{23}z_{23}\|^2 + e_{23}z_{23}^T(\hat{v}_2 - v_0 + e_{12}z_{12}) \\ &\leq -\|e_{13}z_{13} + e_{23}z_{23}\|^2 + \|e_{23}z_{23}\| \|\hat{v}_2 - v_0 + e_{12}z_{12}\| \\ &= -\alpha(z(t)) + \beta(z(t)) \quad (13) \end{aligned}$$

where

$$\begin{aligned} \alpha(z(t)) &:= \|e_{13}z_{13} + e_{23}z_{23}\|^2 \\ \beta(z(t)) &:= \|e_{23}z_{23}\| \|\hat{v}_2 - v_0 + e_{12}z_{12}\| \end{aligned}$$

Theorem 4.5: In the system (2), the distance error and the velocity estimation error of the follower are bounded by applying (12) and (13). (i.e., $|e_{13}|$, $|e_{23}|$ and $\|v_0 - \hat{v}_3\|$ are bounded)

Proof: To investigate boundedness of $|e_{13}|$, $|e_{23}|$ and $\|v_0 - \hat{v}_3\|$, suppose that $\|z_{13}\|$ and $\|z_{23}\|$ are sufficiently large. Because z_{ij} and e_{ij} are defined in (3) and (4), $e_{13} \simeq e_{23} \simeq \|z_{13}\|^2 \simeq \|z_{23}\|^2$ is acquired if $\|z_{13}\|$ and $\|z_{23}\|$ are sufficiently large. Therefore, α behaves as $\Theta\left(\|z_{13}\|^6\right)$.

From Theorem 4.1, it is obtained that $\|\hat{v}_2 - v_0 + e_{12}z_{12}\|$ is bounded and smaller than initial value. Therefore, β behaves as $\Theta\left(\|z_{13}\|^3\right)$.

Here, (13) is rewritten as follows:

$$\dot{V}_3 \leq -\Theta\left(\|z_{13}\|^6\right) + \Theta\left(\|z_{13}\|^3\right)$$

From the relationship $\left|\Theta\left(\|z_{13}\|^6\right)\right| > \left|\Theta\left(\|z_{13}\|^3\right)\right|$ according to their definitions, it can be now concluded that $\dot{V}_3$ is always negative if $\|z_{13}\|$ and $\|z_{23}\|$ are sufficiently large. Therefore, the boundedness of $|e_{13}|$, $|e_{23}|$ and $\|v_0 - \hat{v}_3\|$ is clear. ■

V. SIMULATION

The simulation is performed as depicted in Fig. 2. The initial positions of three agents are given as follows:

$$p_1(0) = (0, 5)\,;\; p_2(0) = (0, 0)\,;\; p_3(0) = (5, 0)$$

and the reference velocity v_0 is given as $v_0 = (10, 10)$. The desired formation is given as:

$$d_{12} = 10;\; d_{13} = 15;\; d_{23} = 15$$

The trajectories of agents are shown in Fig. 3 and the errors of distance are shown in Fig. 4. As a result, it is clear from the figures that the formation is bounded and the desired formation is achieved.

The velocity estimation error of each follower is shown in Fig. 5 and it reveals that the difference between the estimated values (i.e, $\hat{v}_2$ or $\hat{v}_3$) and reference velocity value v_0 converges to zero.

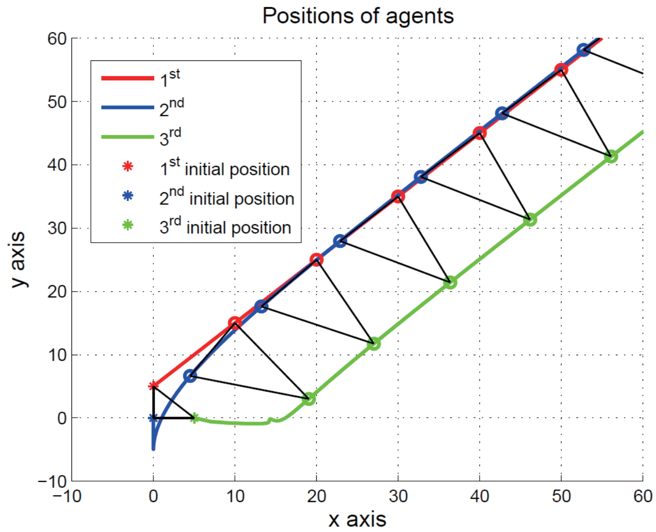

Fig. 3: Trajectories of agents

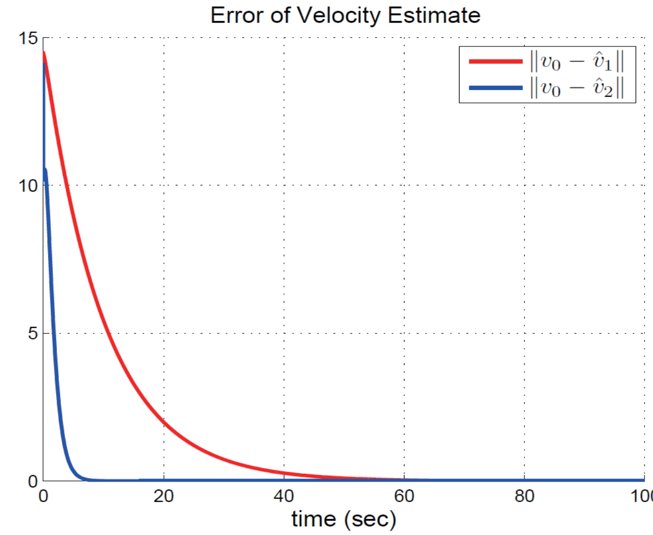

Fig. 5: Errors of velocity estimate

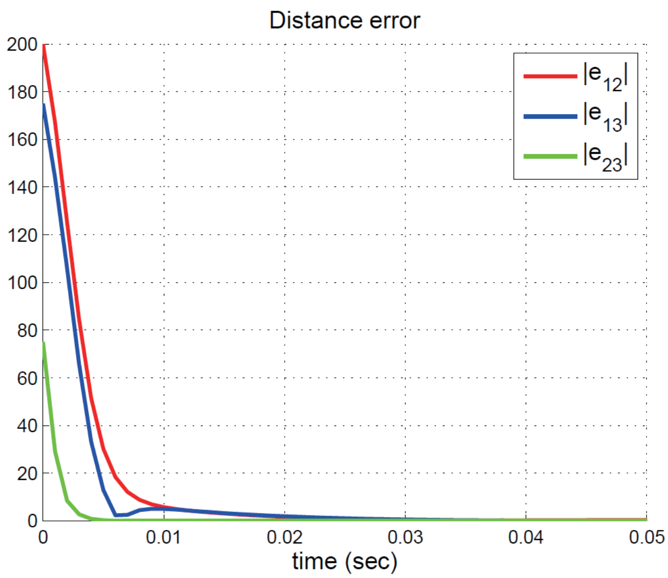

Fig. 4: Errors of distance

VI. CONCLUSIONS

In this paper, the distance-based adaptive formation controller for leader-follower agents system was proposed. From the stability analysis, it was theoretically proved that the proposed controller makes the first follower converge to the desired formation and move with the reference velocity. Further, the follower is stabilized by using the proposed controller. Even though the first follower and the other followers do not know the value of reference velocity, the stability or convergence of system is obviously obtained. By performing the simulation, it is proved that the controller is working as designed.

In our problem setup, the value of reference velocity is restricted to be a constant. In our future works, the problem including time-varying reference velocity shall be considered. Also, we hope to extend the results of this paper to more general dynamics that may have N agents.

VII. ACKNOWLEDGEMENTS

The research of this paper was supported by the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology (No.2011-0021474, No.2013R1A2A2A01067449)

REFERENCES

[1] M. Cao, B. D. O. Anderson, A. S. Morse, and C. Yu, "Control of acyclic formations of mobile autonomous agents," in *47th IEEE Conference on Decision and Control (CDC)*. IEEE, 2008, pp. 1187–1192.

[2] K. K. Oh and H. S. Ahn, "A survey of formation of mobile agents," in *2010 IEEE International Symposium on Intelligent Control (ISIC)*. IEEE, 2010, pp. 1470–1475.

[3] J. Guo, Z. Lin, M. Cao, and G. Yan, "Adaptive leader-follower formation control for autonomous mobile robots," in *American Control Conference (ACC), 2010*. IEEE, 2010, pp. 6822–6827.

[4] Z. Lin, B. Francis, and M. Maggiore, "Necessary and sufficient graphical conditions for formation control of unicycles," *Automatic Control, IEEE Transactions on*, vol. 50, no. 1, pp. 121–127, 2005.

[5] W. Ren and E. Atkins, "Distributed multi-vehicle coordinated control via local information exchange," *International Journal of Robust and Nonlinear Control*, vol. 17, no. 10-11, pp. 1002–1033, 2007.

[6] B. Anderson, C. Yu, S. Dasgupta, and A. Stephen Morse, "Control of a three-coleader formation in the plane," *Systems & Control Letters*, vol. 56, no. 9, pp. 573–578, 2007.

[7] D. V. Dimarogonas and K. H. Johansson, "On the stability of distance-based formation control," in *Decision and Control, 2008. CDC 2008. 47th IEEE Conference on*. IEEE, 2008, pp. 1200–1205.

[8] L. Krick, M. E. Broucke, and B. A. Francis, "Stabilisation of infinitesimally rigid formations of multi-robot networks," *International Journal of Control*, vol. 82, no. 3, pp. 423–439, 2009.

[9] G. Lafferriere, A. Williams, J. Caughman, and J. Veerman, "Decentralized control of vehicle formations," *Systems & control letters*, vol. 54, no. 9, pp. 899–910, 2005.

[10] K.-K. Oh and H.-S. Ahn, "Formation control of mobile agents based on inter-agent distance dynamics," *Automatica*, vol. 47, no. 10, pp. 2306–2312, 2011.

[11] M.-C. Park, K.-K. Oh, and H.-S. Ahn, "Modified gradient control for acyclic minimally persistent formations to escape from collinear position," in *Proceedings of the 51st IEEE Conference on Decision and Control*, Dec. 2012, pp. 1423–1427.

[12] J. Guo, G. Yan, and Z. Lin, "Local control strategy for moving-target-enclosing under dynamically changing network topology," *Systems & Control Letters*, vol. 59, no. 10, pp. 654–661, 2010.

[13] S.-M. Kang, M.-C. Park, B.-H. Lee, and H.-S. Ahn, "Distance-based adaptive formation control for three agents moving with reference velocity," *Submitted for a publication*, vol. -, no. -, pp. –, 2013.
[14] C. Yu, J. M. Hendrickx, B. Fidan, B. D. O. Anderson, and V. D. Blondel, "Three and higher dimensional autonomous formations: Rigidity, persistence and structural persistence," *Automatica*, vol. 43, no. 3, pp. 387–402, 2007.
[15] H. K. Khalil and J. W. Grizzle, *Nonlinear systems*. Prentice hall Upper Saddle River, 2002, vol. 3.