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Finite Time Dynamical Formation Control of Multi-Agent Systems

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Abstract: Finite time dynamic formation control problem of multi-agent systems is considered in this paper. In the dynamic formation problem, the desired formation is dynamic. First, the problem formulation is given in a finite time. Then, the formation control law distanced-based is proposed. Last, the dynamic formation approach is applied to a guidance law design problem, in which multi-flight vehicles cooperative intercept one moving target. A finite time guidance law, which ensure the desired formation achieve and the target be intercepted, is designed.

Key Words: Multi-Agent Systems, Finite Time, Dynamic Formation, Cooperative Interception

1 Introduction

Formation control is an important issue in coordinated control for a group of agents. In many applications, a group of agents are required to maintain a desired spatial pattern. According to different measurements and actively controlled variables, the existing formation control strategies can be classified as position-based methods, displacement-based methods and distance-based methods. In the distance-based control approach, the desired formation shape is specified by a certain set of inter-agent distances, though each agent requires the relative position measurements in order to control the distances.

Graph theory is a natural tool for modeling the multi-agent formation shape. Some of the earliest work to apply graph theory to multi-agent formation can be found in [1], [2], [3], [4],[5]. Rigid graph theory, in particular, naturally ensures that the inter-agent distance constraints of the desired formation are enforced through the rigidity of the underlying graph. This implicitly guarantees that collisions among agents are avoided while acquiring the formation. The concept of graph rigidity is analogous to the rigidity of civil structures. In our case, the ”vertices” of the structure are the agents and the ”bars” connecting the vertices are the inter-robot distance constraints imposed by the desired formation. In this framework, it is convenient to treat each agent as a point and model their motion with the single-integrator equation. Using the inter-agent distances as the controlled variables is that position measurements in a global coordinate frame are not required [6],[7].

Several results dealing with formation control based on the inter-agent distance have appeared in the literature. For the case where the desired formation is planar (2D), formation controllers were proposed in [8],[9],[10],[11],[12] using the single-integrator model for the agents' motion, and the double-integrator model was considered in [13]. The 3D formations using single and double integrator models was considered in [14]. In [15], a 2D formation maneuvering(the agents simultaneously acquire a formation and move cohesively following a given trajectory) control law was presented using the single integrator model for cycle-free persistent formations. In [16], 2D formation maneuvering and target interception schemes were designed using the single-integrator model.

In all the distance-based formation controllers described above, the desired formation is static in the sense that its shape and size are fixed in time. In certain applications, it is necessary that the prescribed formation is dynamic in nature (size and/or shape are time varying) and the dynamic formation need to be achieved in a finite time. The problem of formation control of multi-agent systems where the desired formation is dynamic is considered in [17], but the control law designed is not finite time.

In this paper, we consider the dynamic formation problem in 2D using the single-integrator model and design the finite time control laws. We assume the desired formation is constructed to be infinitesimally and minimally rigid for all time. Based Lyapunov stability theory, we design a formation controller to stabilize the dynamics of the inter-agent distance errors and track the desired formation while avoiding flip ambiguities in a finite time. Then, the dynamic formation approach is applied to a guidance law design problem, in which multi-flight vehicles intercept one uncertain moving target. A finite time guidance law, which ensure the desired formation achieve and the target be intercepted in a finite time, is designed.

The rest of this paper is organized as follows: In Section 2, the concepts of rigid graph theory in $\mathbb{R}^2$ and some preliminaries results are introduced. The problem formulation of finite formation is given in Section 3. In Section 4, we present the finite time dynamic formation control law and stability analysis. Finite time interception problem is considered in Section 5. Finally, concluding remarks future work are made in Section 6.

2 Preliminaries

In this section, in order to describe the problem and make theoretical analysis easily, some concepts of rigid graph theory and some lemmas are introduced.

2.1 Concepts of Rigid Graph Theory

An undirected graph G is a pair (V, E), where $V = \{1, 2, \cdots, n\}$ is the set of vertices, $E \subset V \times V$ is the set of undirected edges such that vertex pair $(i, j) \in E$ then

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$(j,i) \in E$, and $l \in \{1, \cdots, \frac{n(n-1)}{2}\}$ is the number of edges in E. The matrix relating the nodes to the edges is called the incidence matrix $H = \{h_{kj}\} \in \mathbb{R}^{l\times n}$, whose entries are defined as (with arbitrary edge orientations).

$$h_{kj} = \begin{cases} 1, & \text{the } k-th \text{ edge sinks at node } j \\ -1, & \text{the } k-th \text{ edge leaves at node } j \\ 0, & \text{otherwise} \end{cases} \qquad (1)$$

where $k \in \{1, 2, \cdots, l\}$.

Let the set of neighbors of vertex i be

$$\mathcal{N}_i = \{j \in V | (i,j) \in E\} \qquad (2)$$

If $s_i \in \mathbb{R}^2$ is the position of vertex i, then a framework F is a pair (G, s) where $s = (s_1, s_2, \cdots, s_n) \in \mathbb{R}^{2n}$. Notice that a framework is simply a realization of the graph at given points in the plane. Based on an arbitrary ordering of the edges in E, the edge function $\phi : \mathbb{R}^2 \to \mathbb{R}^l$ is given by

$$\phi(s) = (\ldots \|s_i - s_j\|^2 \ldots)^T, \ \ (i,j) \in E \qquad (3)$$

where $\|\cdot\|$ denotes the Euclidean norm. The $k-th$ component of (3), $\|s_i - s_j\|^2$, corresponds to the $k-th$ edge in E connecting vertices i and j. The rigidity matrix will be used to determine the infinitesimal rigidity of the framework, as shown in the following definition. The rigidity matrix $R : \mathbb{R}^2 \to \mathbb{R}^{l\times 2n}$ of $F = (G, s)$ is defined as

$$R(s) = \frac{\partial\phi(s)}{2\partial s}. \qquad (4)$$

It is known that rank $[R(s)] \leq 2n - 3$.

An isometry of $\mathbb{R}^2$ is a bijective map $\mathcal{I} : \mathbb{R}^2 \to \mathbb{R}^2$ such that

$$\|x - y\| = \|\mathcal{I}(x) - \mathcal{I}(y)\|, \forall x, y \in \mathbb{R}^2 \qquad (5)$$

Note that $\mathcal{I}$ accounts for rotation, translation, and reflection of the vector $x - y$. We denote the set of all isometric frameworks of F by Iso(F). It is not difficult to show that (3) is invariant under isometric motions of F.

Two frameworks (G, s) and $(G, \hat{s})$ are equivalent if $\phi(s) = \phi(\hat{s})$ and are congruent if $\|s_i - s_j\| = \|\hat{s}_i - \hat{s}_j\|$ for all $i, j \in V$. We say a framework (G, s) where $n > 2$ and s is generic infinitesimally rigid if and only if rank$[R(s)] = 2n-3$. A framework (G, s) is minimally rigid if $l = 2n-3$. If two infinitesimally rigid frameworks (G, s) and $(G, \hat{s})$ are equivalent but not congruent, then they are said to be flip ambiguous [7]. We denote the set of all flip ambiguities of an infinitesimally rigid framework F and its isometries by Amb(F). We assume that all frameworks in Amb(F) are also infinitesimally rigid. This assumption is reasonable and, in fact, holds almost everywhere which is introduced in [7] and the Theorem 3 of [18].

The preliminary results below will be vital for establishing our main result. Specifically, they will allow us to formalize the stability set of the closed-loop system in relation to the infinitesimal rigidity and flip ambiguities of the framework modeling the formation. To this end, we consider two frameworks $F = (G, s)$ and $\bar{F} = (G, \bar{s})$ sharing the same graph $G = (V, E)$, and the metric

$$\text{dist}(\omega, \mathfrak{M}) = \inf_{x\in\mathfrak{M}} \|\omega - x\| \qquad (6)$$

where ω, $x \in \mathbb{R}^n$. $\mathfrak{M}$ is a set.

2.2 Preliminaries Results

In order to judge infinitesimally rigid, the lemma 1 [16] is given as follow,

Lemma 1 *If F is infinitesimally rigid and* $\text{dist}(\bar{s}, \text{Iso}(F)) \leq \varepsilon$ *where ε is a sufficiently small positive constant, then $\bar{F}$ is also infinitesimally rigid.*

According to the lemma 1, we can have the following lemma [16].

Lemma 2 *Consider the function*

$$\Psi(\bar{F}, F) = \sum_{(i,j)\in E} (\|\bar{s}_i - \bar{s}_j\| - \|s_i - s_j\|)^2 \qquad (7)$$

If F is infinitesimally rigid and $\Psi(\bar{F}, F) \leq \delta$ where δ is a sufficiently small positive constant, then $\bar{F}$ is also infinitesimally rigid.

In order to facilitate the finite-time stability analysis, lemma 3 and lemma 4 are given as follows,

Lemma 3 *For any $x_i \in \mathbb{R}$ $i = 1, 2, \cdots, n$ and a real number* $0 < a \leq 1$.

$$(|x_1| + |x_2| + \cdots + |x_n|)^a \leq |x_1|^a + \cdots + |x_n|^a \qquad (8)$$

Lemma 4 $\dot{x} = f(x, u)$ *is finite-time stable if and only if there is a continuous Lyapunov function V and constants* $K \geq 0$, $0 < a < 1$ *such that*

$$\dot{V} \leq -KV^a \qquad (9)$$

and the finite time

$$T \leq \frac{V^{1-a}(x(0))}{K(1-a)}$$

Based on the definition of the rigidity matrix $R(s)$, the following lemma is given [16],

Lemma 5 *Let $v \in \mathbb{R}^2$ and $\mathbf{1}_n$ be the $n \times 1$ vector of ones, then* $R(s)(\mathbf{1}_n \times v) = 0$.

3 Problem Formulation

In this section, the finite time dynamic formation problem formulation is given. Consider a system of n agents in the plane modeled as,

$$\dot{s}_i = u_i \qquad (10)$$

where $s_i = (x_i, y_i) \in \mathbb{R}^2$ is the $i-th$ agent position with respect to an Earth-fixed coordinate frame. $u_i \in \mathbb{R}^2$ is the control input with respect to an Earth-fixed coordinate frame.

Let the desired formation for the agents be represented by an infinitesimally and minimally rigid framework $F^* = (G^*, s^*)$, $G^* = (V^*, E^*)$, $s^* = (s_1^*, \cdots, s_n^*)$. The time-varying desired distance between agents i and j is given by

$$d_{ij}(t) = \|s_i^*(t) - s_j^*(t)\| > 0, \quad i, j \in V^* \qquad (11)$$

We assume the desired distances d_{ij} and their derivative (with respect to t) $\dot{d}_{ij}$ are bounded, continuous functions and we assume that $\dot{d}_{ij} \to 0$ as $t \to T$, T is the finite time.

Consider that the actual formation of the agents is represented by the framework $F(t) = (G^*, s(t))$ where $s =$

$(s_1, \cdots, s_n)$. Assume that the relative position of agent pairs in E^* can be measured. Further, assume that the agents do not satisfy the desired distance constraints at the initial time $t = 0$, i.e. $\|s_i(0) - s_j(0)\| \neq d_{ij}(0), \quad i, j \in V^*$.

In this paper, finite time dynamic formation and formation maneuvering are considered. The primary control objective for the problem is to design $u_i,\ i \in \{1, 2, \cdots, n\}$ such that

$$F(t) - \text{Iso}(F^*(t)) \to 0 \ \ as\ t \to T \tag{12}$$

Note that (12) is equivalent to

$$\|s_i(t) - s_j(t)\| \to d_{ij}(t), \quad i, j \in V^* \ \ as\ t \to T \tag{13}$$

where T is the finite time, i.e., $T < \infty$.

In the formation maneuvering problem, the objective is

$$u_i \to u_d(t), \quad i \in V^* \ \ as\ t \to T \tag{14}$$

where $u_d \in \mathbb{R}^2$ is the desired translational velocity for the group. We assume u_d is a known C_1 function and u_d is bounded. This assumption is not restrictive because u_d is known a priori and can be pre-stored on each agents onboard computer.

4 Finite Time Dynamical Formation Control

In this section, a finite time control law is given, which makes the desire dynamic formation achieve in a finite time. And the finite time is given.

Define the relative position of two agents as

$$s_i - s_j = \tilde{s}_{ij}(t), \quad (i, j) \in E^* \tag{15}$$

The distance error is given by

$$s_i - s_j = \tilde{s}_{ij}(t), \quad (i, j) \in E^* \tag{16}$$

Let $\tilde{s} = (\cdots, \tilde{s}_{ij}(t), \cdots) \in \mathbb{R}^{2l}$ with the same ordering of terms as (3). The distance error is given by

$$e_{ij} = \|\tilde{s}_{ij}\| - d_{ij} \tag{17}$$

It follows from (16) and (10) that the distance tracking error dynamics is

$$\dot{e}_{ij} = \frac{d}{dt}(\sqrt{\tilde{s}_{ij}\tilde{s}_{ij}}) = \frac{\tilde{s}_{ij}(u_i - u_j)}{e_{ij} + d_{ij}} - \dot{d}_{ij} \tag{18}$$

Consider the potential function

$$W_{ij} = \frac{1}{4}z_{ij}^2, \ \ (i, j) \in E^*$$

where

$$z_{ij}(e_{ij}) = |\tilde{s}_{ij}|^2 - d_{ij}^2 = e_{ij}(e_{ij} + 2d_{ij}) \tag{19}$$

From (19), it is clear that $z_{ij}(e_{ij})$ is only defined on $[-d_{ij}, \infty)$ and $z_{ij} = 0$ if and only if $e_{ij} = 0$. Therefore, W_{ij} is positive definite and radially unbounded in e_{ij}.

We now define the following function

$$W(e) = \sum_{i,j \in E^*} W_{ij}(e_{ij}) \tag{20}$$

where $e = (\cdots, e_{ij} \cdots) \in \mathbb{R}^l \ \ (i, j) \in E^*$, is ordered as (3). The time derivative of (20) along (18) is given by

$$\dot{W}(e) = \sum_{i,j \in E^*} e_{ij}(e_{ij} + d_{ij})[\tilde{s_{ij}}^T(u_i - u_j) - d_{ij}\dot{d}_{ij}] \tag{21}$$

It follows from (31) and (19) that (21) can be rewritten as

$$\dot{W}(e) = z^T(R(s)u - \bar{d}) \tag{22}$$

where $z = (\ldots z_{ij} \ldots)^T \in \mathbb{R}^l$, $u = (u_1, \ldots, u_n)^T \in \mathbb{R}^{2n}$, $\bar{d} = (\cdots, d_{ij}\dot{d}_{ij}, \cdots) \in \mathbb{R}^l$. The elements in z and $\bar{d}$ are ordered in the same way as (3).

Before stating the main result, we give the following lemma which is introduced in [13].

Lemma 6 *For nonnegative constants* c *and* δ*, the level set* $W(e) \leq c$ *is equivalent to* $\Psi(F^*, F) \leq \delta$ *where* Ψ *and* W *were defined in* (7) *and* (20)*, respectively.*

Lemma 6 implies that $W(e) \leq c$ can guarantee that both the desired formation F^* and the actual formation F are the infinitesimally rigid.

The control law for finite time dynamic formation and formation maneuvering is given in the following theorem.

Theorem 1 *Given the formation* $F(t) = (G^*, s(t))$*, let the initial conditions be such that* $e(0) \in \Omega$ *where*

$$\begin{aligned}\Omega = \{e \in \mathbb{R}^l | \Psi(F, F^*) \leq \delta \\ \wedge \text{dist}(s, \text{Iso}(F^*)) \leq \text{dist}(s, \text{Amb}(F^*))\}\end{aligned} \tag{23}$$

and δ *is a sufficiently small positive constant. Then control law*

$$u = R^T(s)(R(s)R^T(s))^{-1}(-k_z z^{2a-1} + \bar{d}) + 1_n \otimes u_d(t) \tag{24}$$

ensures $W(e) \to 0$ *in a finite time, and* (13) *and* (14) *are satisfied, where*

$$z^{2a-1} = (\ldots, (z_{ij})^{2a-1}, \ldots)^T,$$

$k_z > 0$, $\frac{1}{2} < a < 1$.

Proof: First, since $F^*(t)$ and $F(t)$ have necessarily the same number of edges, the minimal rigidity of F^* implies that $F(t)$ is minimally rigid for all $t \geq 0$. Since $F^*(t)$ is infinitesimally rigid, we know from Lemma 2 that $F(t)$ is infinitesimally rigid for $e(t) \in \Omega$. So we have that $F(t)$ is infinitesimally and minimally rigid for $e(t) \in \Omega$. And then $R(s)$ has full row rank and $R(s)R^T(s)$ is invertible for $e(t) \in \Omega$. Therefore, substituting (24) into (21) and applying Lemma 3 yields

$$\dot{W}(e) = -k_z z^T z^{2a-1} \tag{25}$$

From the definition of $(z)^{2a-1}$, we can get

$$z^T(z^{2a-1}) = \sum_{i',j' \in E^*} z_{i'j'}^{2a} \tag{26}$$

According to lemma 3, we get

$$\sum_{i',j' \in E^*} z_{i'j'}^{2a} \geq (\sum_{i',j' \in E^*} z_{i'j'}^2)^a = (z^T z)^a \tag{27}$$

In terms of (27), we have

$$\begin{aligned}\dot{W}(e) &= -k_z z^T z^{2a-1} \\ &\le -k_z(z^T z)^a \\ &\le -4^a k_z(\frac{1}{4}z^T z)^a \\ &= -KW^a\end{aligned} \tag{28}$$

where $K = 4^a k_z$. Applying lemma 4, we have that $W \to 0$ in a finite time and the time is

$$T_1 = \frac{W^{1-a}(e(0)}{K(1-a)} \tag{29}$$

Due to $\dot{d}_{ij} = 0$ as $t \to T_1$, we can have that $\bar{d} = 0$ as $t \to T_1$ and $u \to 1_n \otimes u_d(t)$ that means $u_i \to u_d$ as $t \to T_1$.

Remark 1 *It is not difficult to see that each row of the rigidity matrix $R(s)$ takes the following form*

$$[0, \cdots, (s_i - s_j)^T, 0, \cdots, 0, (s_j - s_i)^T, \cdots, 0] \tag{30}$$

So we have

$$R(s) = \begin{pmatrix} \pi_1^T & 0 & 0 & 0 \\ 0 & \pi_2^T & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \pi_l^T \end{pmatrix} \bar{H}. \tag{31}$$

where $\pi_k^T \in \mathbb{R}^2$ is the relative position vector for the vertex pair defined by the $k-th$ edge, and $\bar{H} = H \bigotimes I_2$.

Remark 2 *Since F is infinitesimally and minimally rigid for $e(t) \in \Omega$, then $R(s)$ has full row rank and $R(s)R^T(s)$ is invertible for $e(t) \in \Omega$. Let*

$$\pi = \begin{pmatrix} \pi_1^T & 0 & 0 & 0 \\ 0 & \pi_2^T & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \pi_l^T \end{pmatrix} \tag{32}$$

so we have

$$R(s)R^T(s) = \pi\bar{H}\bar{H}^T\pi^T \tag{33}$$

and

$$\begin{aligned}R^T(s)(R(s)R^T(s))^{-1} &= \bar{H}^T\pi^T\pi^{-T}(\bar{H}\bar{H}^T)^{-1}\pi^{-1} \\ &= \bar{H}^T(\bar{H}\bar{H}^T)^{-1}\pi^{-1}\end{aligned} \tag{34}$$

The control (24) can be written as follows

$$\begin{aligned}u &= R^T(s)(R(s)R^T(s))^{-1}(-k_z z^{2a-1} + \bar{d}) + 1_n \otimes u_d(t) \\ &= -k_z R^T(s)(R(s)R^T(s))^{-1}(z^{2a-1} + \bar{d}) + 1_n \otimes u_d(t) \\ &= -k_z \bar{H}^T(\bar{H}\bar{H}^T)^{-1}\pi^{-1}(z^{2a-1} + \bar{d}) + 1_n \otimes u_d(t)\end{aligned} \tag{35}$$

5 Cooperative Interception Problem

In this section, the dynamic formation approach is applied to a guidance law design problem, in which multi-flight vehicles cooperative intercept one uncertainly moving target. Because of the uncertainties of the target, only a dynamic region describing the location of target can be obtained. In order to intercept the target, the control strategy for the multi-flight to cover the dynamic region is designed. For the efficiency of the coverage, the dynamic formation strategy is applied. Then a finite time guidance law, which ensure the desired formation achieve and the target be intercepted, is designed.

5.1 Model of The Cooperative Interception Problem

For simplicity, we focus on the interception problem in the plane. In order to describe the problem, a global inertial coordinate system is defined to describe the independent movement of each flight vehicle. The relative motion of an intercepting flight vehicle and a target is described in their line of sight (LOS) coordinate system.

Definition 1 Line of Sight Coordinate System *$O_L x_L y_L$: the mass center of the flight vehicle is selected as the origin O_L, $O_L x_L$ axis is along the connection line of mass center of the target and that of the flight vehicle, pointing to the target; $O_L y_L$ axis is in the vertical plane containing $O_L x_L$ vertical to $O_L x_L$ axis, pointing to the positive direction;*

Definition 2 Global Inertial Coordinate System *Oxy: Select the LOS coordinate system formed by certain target and certain flight vehicle at the initial moment as the global inertial coordinate system. Its a fixed coordinate system with no translation nor rotation.*

In the global inertial coordinate system Oxy, the relative motion between the target and the flight vehicles can be described in Fig. 1. In the Fig. 1, M_i denotes the i-th flight vehicle, T denotes the target, r_i denotes the relative distance between the intercepting flight vehicle M_i and the target T. And V_i denotes the velocity of the intercepting flight vehicle M_i.

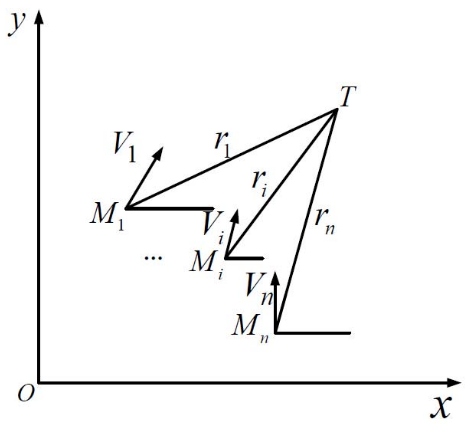

Fig. 1: Planar engagement geometry of terminal guidance control of the flight vehicles

We assume the flight vehicle M_i can get its position information through its inertial devices. Because of the uncertainties of the target, only a dynamic region describing the location of target can be obtained. Let $C(t)$ denote the estimation target region which is time variable with the approaching between the flight vehicle and the target, and $C(t)$ is a circle with radius $R_T(t)$. Assume that the region $C(t)$ can be given by the ground radar. In order to intercept the target, the multi-flight need to cover the target region $C(t)$. For the efficiency of the coverage, one square $\mathcal{D}$ is chose such that its inscribed ball is $C(t)$. And then we divide $\mathcal{D}$ into n small equal squares shown as Fig. 2. For simplicity, denote the centres of the n squares $s^* = (s_1^*, \cdots, s_n^*)$. Then we only need to make n flight vehicles arrive at the centers of these squares, respectively, which means the target region $C(t)$ can be covered. Thus, the coverage problem can be transformed to a dynamic formation problem.

The deployment formation is shown in Fig. 2. In Fig. 2, the blue circle denote the target region $C(t)$, the red square denote the external square $\mathcal{D}$, s_1^*, s_2^*, s_3^* and s_4^* denote the desired position of the flight vehicle $M_k, (k = 1, 2, 3, 4)$, $d_{12}, d_{23}, d_{34}, d_{41}$ and d_{25} denote the desired distance. If the flight vehicle $M_k, (i = 1, 2, 3, 4)$ arrive the desired position s_k^*, the target region $C(t)$ can be covered that means the target can be intercepted.

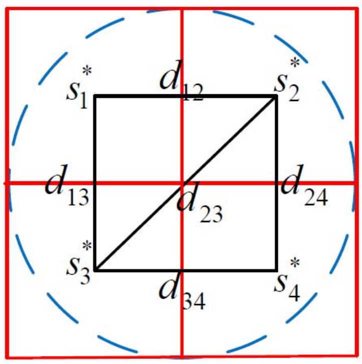

Fig. 2: Formation for coverage tracking

In order to describe the problem easy, we regard the flight vehicle as agent. Let s_i denote the position of the flight vehicle M_i and let s_T denote the position of the target region center in the global inertial coordinate system Oxy. And the relative distance between the s_i and the target region center s_T can be given by the ground radar.

In order to cover the target region $C(t)$, the agents need to track their desired formation. Let the desired formation for the agents be represented by an infinitesimally and minimally rigid framework $F^* = (G^*, s^*)$, $G^* = (V^*, E^*)$, $s^* = (s_1^*, \cdots, s_n^*)$. The actual formation of the n flight vehicles is represented by the framework $F(t) = (G^*, s(t))$ where $s = (s_1, \cdots, s_n)$. Because of the target region is time variable, the desired distance is also time variable. The time-varying desired distance between agents i and j is given by the (11). And the single-integrator model of the flight vehicle M_i is given by (10).

In order to intercept the uncertain moving target, not only dose the target region need to be covered but also the target region center s_T needs to be tracked. We take the $1-th$ agent to be the leader while the remaining agents are followers. Our control scheme will consist of: a) selecting the infinitesimally and minimally rigid framework F^* such that $s_1^* \in \text{conv}\{s_2^*, s_3^*, \cdots, s_n^*\}$, where $\text{conv}\{\cdot\}$ denotes the convex hull, b) the leader tracking the target region centre in a finite time, and c) the followers tracking the leader while maintaining the desired formation in a finite time. Thus, the objective for this problem is

$$s_T \in \text{conv}\{s_2 \cdots, s_n\} \ \ as \ \ t \to T \tag{36}$$

where $s_T \in \mathbb{R}^2$ denotes the target region centre position and $T < \infty$. In the analysis above we have known that $s_1 - s_T$ can be given by ground radar, and we assume that $\dot{s}_T$ is bounded.

5.2 Finite time Guidance Law Design

In this subsection, the finite time guidance law is designed. To this end, we define the target region centre's relative position to the leader (i.e., target interception error) as

$$e_T = s_1 - s_T \tag{37}$$

And $\dot{s}_T \triangleq v_T$. Assume the velocity of all the point in the target region $C(t)$ is same, so the velocity of the target region is also v_T.

In general, the velocity of the target region centre is unknown. In this case, the agents need to estimate the velocity of the target region centre. In this paper, we do not give the estimation method of the velocity of the target region centre, but we give the following assumptions,

Assumption 1 $\|\hat{v}_T - v_T\| \leq \zeta$ *where* $\hat{v}_T$ *is the estimation of the velocity of the target region centre, and* ζ *is a sufficiently small positive constant.*

Assumption 2 $\|e_T\| \leq \gamma$ *where* γ *is a positive constant.*

Then, a lemma is introduced as follows.

Lemma 7 *Consider the system* $\dot{x} = f(x, u)$. *Suppose that there exist continuous function* $V(x)$, *scalars* $\lambda > 0$, $0 < \sigma < 1$ *and* $0 < \beta < \infty$ *such that*

$$\dot{V}(x) \leq -\lambda V(x)^\sigma + \beta \tag{38}$$

Then, the trajectory of system $\dot{x} = f(x, u)$ *is PFS(Practical Finite-time Stable),and the trajectories of the closed-loop system is bounded in finite time as*

$$\lim_{t\to T} x \in (V^\sigma(x) \leq \frac{\beta}{(1-\theta)\lambda}) \tag{39}$$

where $0 < \theta < 1$. *And the time needed to reach (39) is bounded as*

$$T \leq \frac{V^{1-\sigma}(x(0))}{(1-\sigma)\theta\lambda}) \tag{40}$$

Lastly, the theorem below gives the second main result of this section.

Theorem 2 *Based on the Assumption 1 and Assumption 2, the actual formation of the* n *flight vehicles is given as* $F(t) = (G^*, s(t))$, *and let the initial conditions be such that* $e(0) \in \Omega$ *where*

$$\begin{aligned} \Omega = \{&e \in \mathbb{R}^l | \Psi(F, F^*) \leq \delta \\ &\wedge \text{dist}(s, \text{Iso}(F^*)) \leq \text{dist}(s, \text{Amb}(F^*))\} \end{aligned} \tag{41}$$

and δ *is a sufficiently small positive constant. Then guidance law*

$$u = R^T(s)(R(s)R^T(s))^{-1}(-k_z z^{2a-1} + \bar{d}) + h, \tag{42}$$

where

$$h = 1_n \otimes (\hat{v}_T - k_T e_T^a) \tag{43}$$

and $\frac{1}{2} < a < 1$, *ensures* $W(e) \to 0$ *in a finite time, and* (13) *and* (36) *are satisfied.*

Proof: The proof of Theorem 1 can be followed to show that, for $e(0) \in \Omega$, (42) ensures that $e = 0$ in a finite time that means (13) holds. In terms of (42), (43), and (10), we have

$$\dot{s} = 1_n \otimes (\hat{v}_T - k_T e_T^a) \ \ as \ \ t \to T_1 \quad (44)$$

It means that

$$\dot{s}_i = \hat{v}_T - k_T e_T^a \ \ as \ \ t \to T_1 \quad (45)$$

and the derivative of (37) with respect to time t is

$$\dot{e}_T = \dot{s}_1 - \dot{s}_T = -k_T e_T^a + r \quad (46)$$

where $r = \hat{v}_T - \dot{s}_T$. Selecting Lyapunov as

$$W_T(e_T) = \frac{1}{2} e_T^T e_T \quad (47)$$

The derivative for $W_T(e_T)$ with respect to t, and according to (46) we have

$$\dot{W}_T(e_T) = e_T^T \dot{e}_T = -k_T e_T^T e_T^a - k_T e_T^T r \quad (48)$$

According to Assumption 1 and Assumption 2, we know that

$$\dot{W}_T(e_T) \leq -2k_T W_T(e_T)^{\frac{1+a}{2}} + k_T \zeta\gamma. \quad (49)$$

According to lemma 7, we have that e_T converges a bounded set, as $t \to T_2$. And the finite time is

$$T_2 \leq \frac{W_T^{\frac{1-a}{2}}(e_T(0))}{(1-a)\theta k_T}$$

Note that e_T converges to the bounded set in finite time as

$$\lim_{t \to T_2} e_T \in (W_T^{\frac{1+a}{2}}(e_T) \leq \frac{k_T \zeta\gamma}{(1-\theta)2k_T}) \quad (50)$$

It means that target intercepting error includes a constant offset so that the leader does not end up on top of the target, i.e., $e_T = s_1 - s_T \to \eta$, as $t \to T_2$, $\eta \in \mathbb{R}^2$ and η is associated with $\frac{k_T \zeta\gamma}{(1-\theta)2k_T}$. In this case, (36) are satisfied in a finite time.

Therefore, it means that (13) and (36) are satisfied in a finite time. The finite time is $T = \max\{T_1, T_2\}$.

6 Concluding Remarks and Future Work

Finite time dynamic formation control problem of multi-agent systems is considered in this paper. First, the problem formulation is given in a finite time. Then, the formation control law distanced-based is proposed. Last, the dynamic formation approach is applied to a guidance law design problem, in which multi-flight vehicles intercept one moving target. Because of the uncertainties of the target, only a dynamic region describing the location of target can be obtained. In order to intercept the target, the multi-flight need to cover the target region. For the efficiency of the coverage, the dynamic formation strategy is applied. In fact, our idea is to transform the coverage problem to a dynamic formation problem. The case that the velocity of target region centre is unknown is considered in this paper. A finite time guidance law, which ensure the desired formation achieve and the target be intercepted, is designed for this case.

In current work, the dynamic formation for singer-order model of mulit-agent systems were considered. In future work, we will be devoted to extending the proposed approach to account for the agent (vehicle) dynamics for second-order model of mulit-agent systems.

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